

Inflation, moduli (de)stabilization and supersymmetry breaking

Tatsuo Kobayashi^{1,*} and Manabu Sakai^{2,†}

¹*Department of Physics, Kyoto University, Kyoto 606-8502, Japan*

²*Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

Abstract

We study the cosmological inflation from the viewpoint of the moduli stabilization. We study the scenario that the superpotential has a large value during the inflation era enough to stabilize moduli, but it is small in the true vacuum. This scenario is discussed by using a simple model, one type of hybrid models.

*email: kobayash@gauge.scphys.kyoto-u.ac.jp

†email: msakai@yukawa.kyoto-u.ac.jp

1 Introduction

Moduli fields play an important role in superstring theory. Couplings in 4D low-energy effective field theory are given as functions of vacuum expectation values (VEVs) of moduli fields. Thus, we need a stabilization mechanism of moduli VEVs. Indeed, moduli stabilization is one of important issues in string phenomenology and cosmology. The moduli potential has a small bump, which is related to the gravitino mass $m_{3/2}$, in many models of moduli stabilization, e.g. the Kachru-Kalosh-Linde-Trivedi (KKLT) potential [1] and the racetrack potential [2].

When we consider the low-energy supersymmetry (SUSY) breaking scenario, the height of the above bump is quite low. That may lead to some problems. For example, we need the positive energy to derive inflationary expansion of the Universe. If such an inflation energy is larger than this small bump of the moduli potential, the moduli would be destabilized and run away to infinity. Thus, we may have a constraint between the gravitino mass $m_{3/2}$ and the Hubble constant H_{inf} during inflation, e.g. $m_{3/2} \geq H_{inf}$ in a simple model [3].

One way out is to use non-perturbative superpotential with positive exponents [4, 5, 6], although the KKLT superpotential and the usual racetrack superpotential include non-perturbative terms with negative exponents. However, the positive exponents are possible and have significant effects in the moduli stabilization. That is, the moduli potential with positive exponents could have a quite high barrier, which is independent of the gravitino mass.

In this paper, we study another scenario to stabilize the moduli during the inflation era and to lead to low-energy SUSY. We consider the inflation scenario that the inflaton field ϕ , which is different from moduli, drives the inflation dominantly. We assume that the would-be inflaton ϕ induces a large value of superpotential $\langle W(\phi) \rangle_{inf}$ during the inflation era. A similar idea has been studied in [7]. At any rate, such a large value of $\langle W(\phi) \rangle_{inf}$ could also induce a large mass of the modulus during the inflation era, and the modulus mass during the inflation would be of $\mathcal{O}(10) \times \langle W(\phi) \rangle_{inf}$, e.g. for the KKLT superpotential. Here we use the unit that $M_{Pl} = 1$, where M_{Pl} denotes the Planck scale. If such a mass is larger than the Hubble parameter H_{inf} during the inflation era, the modulus is not be destabilized.¹ After the inflation ends, we assume that a small value of the gravitino mass $m_{3/2}$, i.e. $m_{3/2} \ll \langle W(\phi) \rangle_{inf}$, is realized at the potential minimum. At such a potential minimum, the modulus has a large mass compared with the gravitino mass $m_{3/2}$ such as $\mathcal{O}(10) \times m_{3/2}$ or more. Then, the modulus is stabilized at the true minimum. That is the realization of the F-term uplifting scenario at the potential minimum [8, 9, 10, 11].

To illustrate our scenario, we use the inflation model studied in Ref. [14], which is a kind of hybrid inflation models [12, 13]. Its inflaton superpotential $W(\phi)$ is the Intriligator-Seiberg-Shih (ISS) type of superpotential [15] with a deformation proposed in Ref. [16]. (See also for the inflation model with the ISS superpotential [17].) With such an inflation superpotential $W(\phi)$, we study the modulus behavior during the inflation era

¹ This behavior is similar to the F-term uplifting scenario [8, 9, 10, 11].

for the KKLT superpotential and the racetrack superpotential including the case with a positive exponent. Indeed, similar superpotential models have been studied for inflation models [18]. However, our viewpoint differs from those.

This paper is organized as follows. In section 2, we explain our scenario without explicit inflation superpotential. In section 3, we study our scenario by using an illustrating model. Section 4 is devoted to conclusion and discussion.

2 Scenario

Here, we outline our scenario. We assume a superpotential $W(\phi)$ for the would-be inflaton ϕ , which induces a large Hubble parameter H_{inf} in the inflation era (without taking into account the degree of freedom of the moduli). Then, we study the degree of freedom of the moduli. For simplicity, we consider a single modulus, T , with its Kähler potential,

$$-3\ln(T + T^*). \quad (1)$$

Then, the total Kähler potential is written by

$$K = -3\ln(T + T^*) + K(\phi, \phi^*), \quad (2)$$

where the second term denotes the Kähler potential of ϕ and a simple model may correspond to the canonical form, $K(\phi, \phi^*) = |\phi|^2$. The modulus T cannot be stabilized without non-perturbative terms. Thus, we introduce a certain non-perturbative term. First, we consider the KKLT-type superpotential. That is, the total superpotential is written by

$$W = W(\phi) - Ae^{-aT}, \quad (3)$$

with $a > 0$. Here, we assume $a = \mathcal{O}(10)$, which is a natural value when this non-perturbative term is induced e.g. by gaugino condensation. Then, the total scalar potential is written as

$$V = e^K [|D_I W|^2 (K^{I\bar{I}}) - 3|W|^2], \quad (4)$$

where $D_I W = K_I W + W_I$ and $K^{I\bar{I}}$ denotes the inverse of the Kähler metric $K_{I\bar{I}}$.

For the moment, we neglect the dynamics of ϕ . That is, we replace ϕ by a fixed value and neglect $D_\phi W$, i.e. $D_\phi W = 0$ in the scalar potential. In such a case, the modulus T can be stabilized at $D_T W = 0$, i.e.,

$$K_T(W(\phi) - Ae^{-aT}) + aAe^{-aT} = 0, \quad (5)$$

where $W(\phi)$ is considered as a constant. When $|aT| \gg 1$, this equation becomes

$$aT \approx -\log(W(\phi)) - \log(a(T + T^*)/3), \quad (6)$$

and we obtain $|W(\phi)| \gg |Ae^{-aT}|$, that is, the term $W(\phi)$ is dominant in the superpotential. To realize $|aT| \gg 1$, the superpotential $W(\phi)$ must be small such as $|W(\phi)| \ll 1$. At this point, the modulus has a mass

$$m_T = e^{k/2}(K^{TT*})W_{TT} = a^2 A e^{-aT} e^{k/2}(K^{TT*}) \approx \frac{a}{\sqrt{T+T^*}}W(\phi). \quad (7)$$

Now, let us take into account the dynamics of ϕ , that is, non-vanishing $D_\phi W$. We assume that through non-vanishing $D_\phi W$ the inflaton field ϕ induces a large positive energy to drive the inflation. That is, a large Hubble parameter H_{inf} is induced. Such a large Hubble parameter may destabilize the modulus and the modulus may run away to infinity. Once the modulus is destabilized in such a way, the successful inflation cannot be realized. However, the modulus has the mass $m_T \approx aW(\phi)/\sqrt{T+T^*}$ at the above point (6). Thus, the modulus would be stabilized still almost at the above point (6), when such a modulus mass is large enough compared with the Hubble parameter, i.e. $m_T \gg H_{inf}$. Then, the successful inflation could be realized. More precisely speaking, the value of T is changing during inflation as the would-be inflaton ϕ is moving along a slow-roll direction. Hence, the true inflaton is a linear combination of T and ϕ . However, the ϕ direction is dominant in the true inflaton because of the large mass m_T of T . The keypoint to realize our scenario is the inflation model with a large value of $W(\phi)$ such that $aW(\phi)/\sqrt{T+T^*} \gg H_{inf}$ during the inflation era. After the inflation, the system approaches toward the true minimum. We assume that at the true minimum a small gravitino mass, $m_{3/2} \ll \langle W(\phi) \rangle_{inf}$, and low-energy SUSY breaking are realized.

Figures 1 and 2 show what we expect in our scenario. That is, the would-be inflaton ϕ has a flat potential with a large energy corresponding to H_{inf} in a certain region, and that would drive the inflation. In the same region, the value of superpotential W_{inf} is also large and it induces a large mass along the modulus direction like $m_T = \mathcal{O}(10)W_{inf}$. After the inflation ends, the system approaches toward the true vacuum, which corresponds to $\phi = 0$ in these figures. At that point, we would have a gravitino mass smaller than W_{inf} and H_{inf} .

Furthermore, we comment on the superpotential with a positive exponent, e.g.

$$W = W(\phi) + Ae^{aT}, \quad (8)$$

with $a > 0$. Such a non-perturbative superpotential may occur by gaugino condensation, where its gauge kinetic function f is a linear combination between S and T ,

$$f = mS - wT, \quad (9)$$

with $m, w > 0$, and the field S is stabilized already by its heavy mass. (See for details [4, 5].) In this case, we may have $A \ll 1$ in Eq. (8). With this superpotential, the modulus T has the mass $m_T = aW(\phi)/\sqrt{T+T^*}$ around $D_TW = 0$ like Eq. (7). Then, when $m_T \gg H_{inf}$, the successful inflation would be realized as the case with the superpotential (8). The modulus mass m_T is the same between the superpotentials with the negative exponent (3) and positive exponent (8). In both superpotentials, the successful inflation

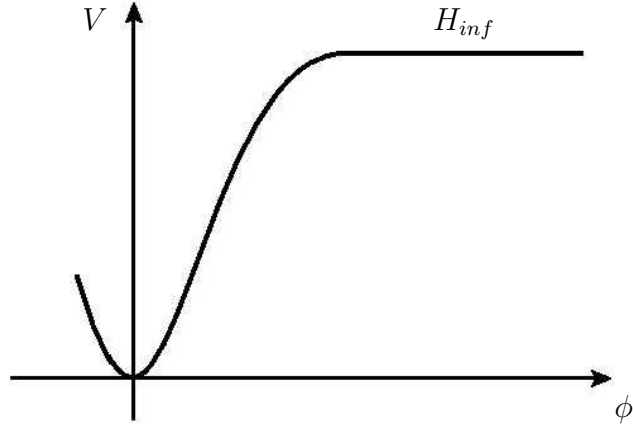


Figure 1: The scalar potential

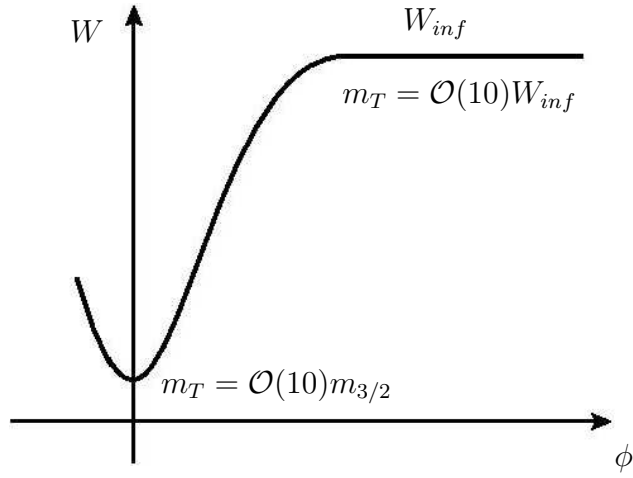


Figure 2: The superpotential

would occur similarly. However, the forms of those scalar potentials are different from each other as we will show later by an explicit form of $W(\phi)$. We can also consider the following superpotential

$$W = W(\phi) + Ae^{-aT} + Be^{bT}, \quad (10)$$

with $a, b > 0$, where we may have $B \ll 1$ similar to A in Eq. (8).

3 A simple model

Here, we illustrate our scenario by using a simple model. As such an illustrating model for the inflation superpotential $W(\phi)$, we use the deformed ISS inflation [14]. (See also [17].)

3.1 Deformed ISS inflation

First, we briefly review on the (deformed) ISS inflation model. This model is the supersymmetric $SU(N)$ gauge theory with N_f flavors of fundamental and anti-fundamental matter fields, q_i^c and \bar{q}_j^c and singlets, M_{ij} , where $i, j = 1 \dots N_f, c = 1 \dots N$. Their superpotential is written by

$$W_{ISS} = w_0 + hq_i^c M_{ij} \bar{q}_j^c - m^2 M_{II} - \mu^2 M_{aa}, \quad (11)$$

where $I = 1 \dots N, a = N + 1 \dots N_f$ and w_0 is a constant. We assume

$$w_0 \sim \mu^2 \ll m^2 \ll 1, \quad (12)$$

and these fields have canonical Kähler potential:²

$$K_{ISS} = |M_{ij}|^2 + |q_i^c|^2 + |\bar{q}_i^c|^2. \quad (13)$$

Now, let us decompose fields as follow:

$$M_{ij} = \begin{pmatrix} Y_{IJ} & Z_{Ia} \\ \bar{Z}_{aI} & \Phi_{ab} \end{pmatrix}, \quad q_i^c = (\chi_I^c \quad \rho_a^c), \quad \bar{q}_i^c = \begin{pmatrix} \bar{\chi}_I^c \\ \bar{\rho}_a^c \end{pmatrix}. \quad (14)$$

The SUSY conditions, $D_I W = 0$, for all fields cannot be satisfied by the rank condition. The minimum corresponds to the point, which satisfies the conditions, $D_\chi W = D_\rho W = D_Z W = D_Y W = 0$, that is,

$$\rho = \bar{\rho} = Z = \bar{Z} = 0, \quad hY = -W, \quad \chi = \bar{\chi} = \sqrt{m^2/h + |Y|^2}, \quad (15)$$

² If higher order terms are large, this model would not satisfy the slow-roll conditions. It is studied that the higher order terms are small enough in the (deformed) ISS inflation [17, 14]. At any rate, we are using the deformed ISS inflation as an illustrating model for our scenario.

but does not satisfy $D_\Phi W = 0$. That is the so-called ISS vacuum. Thus, at such a vacuum the SUSY is broken by F-terms of Φ_{ab} . We may tune the constant w_0 such that the almost vanishing vacuum energy, $V_0 \sim 0$, at this vacuum is realized after including the modulus potential.

Here, we study the inflation in this potential. Indeed, in this model, the would-be inflaton ϕ corresponds to the diagonal direction of Y_{IJ} , i.e., $Y_{IJ} = \frac{1}{\sqrt{N}}\phi\delta_{IJ}$ [14]. In the global SUSY limit, the relevant term of scalar potential can be written as

$$V_{tree} \simeq \left| m^2 \sqrt{N} - \frac{h}{\sqrt{N}} \chi \bar{\chi} \right|^2 + \frac{h^2}{N} |\phi|^2 (|\chi|^2 + |\bar{\chi}|^2), \quad (16)$$

at the tree-level, where we have also neglected μ . From now on, we concentrate ourselves on only the real part of ϕ . Along $\chi = \bar{\chi} = 0$, the field ϕ has the almost flat potential. When $\phi > \phi_c \equiv \sqrt{\frac{N}{h}}m$, the fields χ and $\bar{\chi}$ become massive. Then, ϕ field comes to have the following potential by integrating out $\chi, \bar{\chi}$:

$$V_{eff}(\phi) = Nm^4 + \frac{N^2}{32\pi^2} \left[2h^2m^4 \log \left(\frac{h^2\phi^2}{NM_*^2} \right) + \left(\frac{h^2}{N}\phi^2 + hm^2 \right)^2 \log \left(1 + \frac{Nm^2}{h\phi^2} \right) + \left(\frac{h^2}{N}\phi^2 - hm^2 \right)^2 \log \left(1 - \frac{Nm^2}{h\phi^2} \right) \right], \quad (17)$$

where M_* denotes a cut-off scale and we take $M_* = M_{Pl}$. By this effective potential, the inflaton ϕ now rolls over. When $\phi = \phi_c \equiv \sqrt{\frac{N}{h}}m$, the scalar fields $\chi, \bar{\chi}$ become tachyonic, and roll off to the vacuum with non-vanishing VEVs of χ and $\bar{\chi}$. Then, the inflationary process ends and the system approaches toward the ISS vacuum, which was said above.

In the above analysis, we have taken the global SUSY limit. When we consider the supergravity scalar potential derived from the Kähler potential K_{ISS} and the superpotential W_{ISS} , the mass term of $TrY = \sqrt{N}\phi$ has a correction of $\mathcal{O}(\mu^2/M_{Pl})$. Such a correction can be negligible compared with the above 1-loop correction when μ is sufficiently small.

In this model, the Hubble parameter during the inflation era is evaluated as

$$H_{inf} = \frac{V^{1/2}}{\sqrt{3}M_{Pl}} \sim \frac{\sqrt{N}m^2}{\sqrt{3}M_{Pl}}. \quad (18)$$

Furthermore, the superpotential during the inflation era is dominated by TrY , i.e.,

$$\langle W \rangle_{inf} \sim m^2 TrY. \quad (19)$$

These values are important as said in the previous section, and we will study them explicitly in the following section. In this model, the sizes of the Hubble parameter H_{inf} and the SUSY breaking are determined by the parameters, m^2 and μ^2 , respectively. They are independent each other. In addition, we are assuming $\mu^2 \ll m^2$ in Eq. (12). Thus, we

could realize the inflation with the low-energy SUSY breaking scenario, when the moduli are stabilized successfully.

Here, let us estimate the slow-roll parameters, ϵ and η . We now parameterize the inflaton trajectory by $x \equiv \frac{\phi}{\phi_c}$, which leads to the effective potential of the form,

$$V_{eff}(x) = Nm^4 \left[1 + \frac{Nh^2}{32\pi^2} \left[2 \log \left(\frac{hm^2 x^2}{M_*^2} \right) + (x^2 + 1)^2 \log \left(1 + \frac{1}{x^2} \right) + (x^2 - 1)^2 \log \left(1 - \frac{1}{x^2} \right) \right] \right]. \quad (20)$$

Then, the slow-roll parameters are given by

$$\begin{aligned} \epsilon &= \frac{M_{Pl}^2}{2} \left(\frac{1}{V} \frac{\partial V}{\partial \phi_1} \right)^2 \simeq \frac{h^5 N M_{Pl}^2}{128\pi^4 m^2} x^2 \left[(x^2 - 1) \log \left(1 - \frac{1}{x^2} \right) + (x^2 + 1) \log \left(1 + \frac{1}{x^2} \right) \right]^2, \\ \eta &= M_{Pl}^2 \frac{1}{V} \frac{\partial^2 V}{\partial \phi_1^2} \simeq \frac{h^3 M_{Pl}^2}{8\pi^2 m^2} \left[(3x^2 - 1) \log \left(1 - \frac{1}{x^2} \right) + (3x^2 + 1) \log \left(1 + \frac{1}{x^2} \right) \right]. \end{aligned} \quad (21)$$

We assume that the inflation takes place in the TrY direction, even when we include the modulus T . As shown in [14], the number of e-foldings, $N_e \sim 54 \pm 7$ and the amplitude of curvature perturbation, $P_\zeta^{1/2} = 4.86 \times 10^{-5}$ are obtained as follow

$$N_e \simeq 4\pi^2 \left(\frac{m}{M_{Pl}} \right)^2 \frac{x_e^2}{h^3}, \quad P_\zeta^{1/2} \simeq \frac{1}{\sqrt{2\epsilon}} \left(\frac{H_{inf}}{2\pi M_{Pl}} \right), \quad (22)$$

where x_e denotes $x_e = \phi_e/\phi_c$ with the initial value of the inflaton ϕ_e . Inserting the approximate expressions of these parameters, $H_{inf} = \sqrt{V/3M_{Pl}^2} \simeq \frac{\sqrt{Nm^2}}{\sqrt{3}M_{Pl}}$ and $\epsilon \simeq \frac{h^2}{32\pi^2} \frac{N}{N_e}$, we can obtain

$$\frac{m}{M_{Pl}} \sim 10^{-3} \times h^{1/2}, \quad (23)$$

for $N_e \sim 54$.

3.2 Modulus stabilization and inflation

Here, we consider the following Kähler potential and the superpotential,

$$K = -\ln(T + T^*) + K_{ISS}, \quad W = W_{ISS} - Ae^{-aT}. \quad (24)$$

First we study the ISS vacuum taking account the modulus T . At the point, the dominant terms in the superpotential can be written as

$$W_{ISS} = w_0 - \mu^2 \Phi_{aa} + \dots. \quad (25)$$

At this minimum, the SUSY is broken by F-terms of Φ_{aa} , i.e. $|F^\Phi| = \mu^2$. We assume that the VEV of Φ at the ISS vacuum is small. Then, the VEV of W_{ISS} is dominated by w_0 . The gravitino mass $m_{3/2}$ is evaluated as $m_{3/2} \simeq w_0/(T + T^*)^{3/2}$. When we neglect F^Φ , the modulus T is stabilized at $D_TW = 0$, where the modulus has the mass,

$$m_T \approx \frac{a}{\sqrt{T + T^*}} W_{ISS} \approx \frac{aw_0}{\sqrt{T + T^*}}. \quad (26)$$

Next, we take into account the effect of non-vanishing F^Φ . However, when $F^\Phi = \mu^2$ is smaller than the above modulus mass m_T , the modulus is stabilized almost around the point, $D_TW = 0$. Indeed, we fine-tune w_0 to lead almost vanishing vacuum energy, $V_0 \sim 0$ [10],

$$w_0 - Ae^{-aT} \simeq \frac{1}{\sqrt{3}}\mu^2 \left(1 - \frac{2}{3} \left(\frac{\mu}{m_\Phi} \right)^2 \mu^2 \right), \quad (27)$$

where m_Φ is the mass of Φ at the ISS vacuum. That is the F-term uplifting scenario [9, 10]. Thus, the modulus mass is heavier than the gravitino mass by the factor a . The minimum of T is slightly shifted from the point $D_TW = 0$ and the non-vanishing F^T appears as $F^T \simeq 3a^{-1}m_{3/2}$. At any rate, we have $F^T \ll F^\Phi = \mu^2$. The gravitino mass at the ISS vacuum is evaluated as

$$m_{3/2} \simeq \frac{\mu^2}{\sqrt{3}(T + T^*)^{3/2}} \left(1 - \frac{2}{3} \left(\frac{\mu}{m_\Phi} \right)^2 \mu^2 \right). \quad (28)$$

Now, let us study the inflation with taking into account the modulus T . During the inflation era, the first term W_{ISS} in Eq. (24) becomes large as Eq. (19). Then, for a fixed value of TrY , the modulus T is stabilized around $D_TW = 0$, i.e.

$$aT \sim -\log(m^2 TrY), \quad (29)$$

as studied in Eqs. (5) and (6) of section 2. Around that point, the modulus mass m_T is estimated by Eq. (7) as

$$m_T \approx \frac{a}{\sqrt{T + T^*}} W_{ISS} \approx \frac{am^2}{\sqrt{T + T^*}} TrY. \quad (30)$$

On the other hand, the Hubble parameter during the inflation era is evaluated as

$$H_{inf} = \frac{1}{\sqrt{3}}V^{1/2} \sim \frac{\sqrt{N}m^2}{\sqrt{3}(T + T^*)^{3/2}}. \quad (31)$$

Thus, the modulus is not destabilized during the inflation era if

$$a(T + T^*) TrY > \sqrt{N/3}. \quad (32)$$

For example, when $a(T + T^*) \sim 50$ and $\sqrt{N/3} \sim 1$, this condition is satisfied for $TrY \geq \mathcal{O}(0.1)$. Such a value of T , i.e. $a(T + T^*) \sim 50$ would be obtained for $m \sim 10^{14}$ GeV. Then, a sufficiently large value of e-folding N_e can be realized during $TrY \geq \mathcal{O}(0.1)$ when the initial condition of ϕ is large enough, e.g. $\phi_e = 1$. The magnitude of the Hubble parameter H_{inf} during the inflation is determined by m^2 in Eq. (31), while the gravitino mass $m_{3/2}$ at the ISS vacuum is determined by μ^2 in Eq. (28). Recall that μ^2 and m^2 are independent each other and we are assuming $\mu^2 \ll m^2$ in Eq. (12). Thus, we would realize much smaller value of $m_{3/2}$ than H_{inf} .

When we include the term Ae^{-aT} in the superpotential and $-3\ln(T+T^*)$ in the Kähler potential, the supergravity scalar potential has a correction on the would-be inflaton mass and it is of $\mathcal{O}(Ae^{-aT}/(T+T^*)^{3/2})$, which is estimated as

$$Ae^{-aT}/(T+T^*)^{3/2} \approx \frac{3m^2 TrY}{a(T+\bar{T})^{5/2}}. \quad (33)$$

Such a correction can be neglected for the inflation dynamics because $a = \mathcal{O}(10)$.

Figure 3 shows an example of the scalar potential V of ReT and ϕ . Figure 4 shows the same scalar potential V of ReT at the slice of $\phi = 1.0$ (yellow curve), $\phi = 0.8$ (red curve) and $\phi = 0.3$ (green curve). The inflation continues as far as the modulus is stabilized at the local minimum around $ReT \sim 1.6$, e.g. for $\phi = 1.0$ (yellow curve) and $\phi = 0.8$ (red curve) as well as smaller value of ϕ . (We will study later the point corresponding to $\phi = 0.3$ (yellow curve).) We may need fine-tuning to take the initial condition of T as $ReT \sim 1.6$.

We study the final stage of the inflation. At least, the waterfall point, $\phi_c = \sqrt{\frac{N}{h}}m$ should be larger than the point of ϕ leading to $m_T \sim H_{inf}$, since the moduli is destabilized at $m_T \sim H_{inf}$. We denote a value of the would-be inflaton by $\phi_*(\equiv gM_{Pl}, g < 1)$, where the modulus direction becomes tachyonic. At $\phi = \phi_*$, we would have

$$m_T \approx \frac{am^2}{\sqrt{T+T^*}} TrY \sim H_{inf}. \quad (34)$$

Then, the condition that the modulus is not destabilized during the whole inflation process is written by

$$\phi_* < \phi_c < \phi_e. \quad (35)$$

We assume that the initial value of the would-be inflaton is M_{Pl} , i.e. $\phi_e = M_{Pl}$. Then, for $N_e \sim 54$, we obtain $Nh^2 = \frac{4\pi^2}{N_e} \sim 0.73$ and $\phi_e = 10^{-3}\sqrt{N}M_{Pl}$, where we used Eq. (22). The above condition (35) becomes

$$10^6 g^2 < N < 10^6 \quad \text{or} \quad 10^{-3} < h < 10^{-3} g^{-1}. \quad (36)$$

In our model, a value of g is of $\mathcal{O}(0.1)$, then we need a large number of $N \sim \mathcal{O}(10^5)$.

At any rate, when the above situation is satisfied, the waterfall occurs at $\phi = \phi_c$ by the tachyonic masses of χ and $\bar{\chi}$ directions. During such a waterfall the χ and $\bar{\chi}$ develop their values, which we parameterize them as $\chi\bar{\chi} = \alpha m^2/h$, where α varies $\alpha = 0 - 1$.

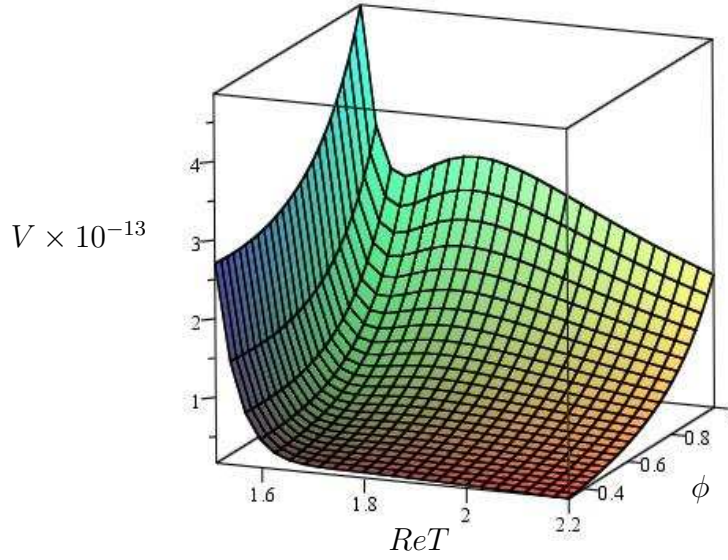


Figure 3: The potential as a function of the moduli ReT and the would-be inflaton ϕ . Values of the parameters are taken as $A = 1, a = 10, w_0 = -10^{-14}, N = 3, m = 7.0 \times 10^{-4}, h = 0.5$.

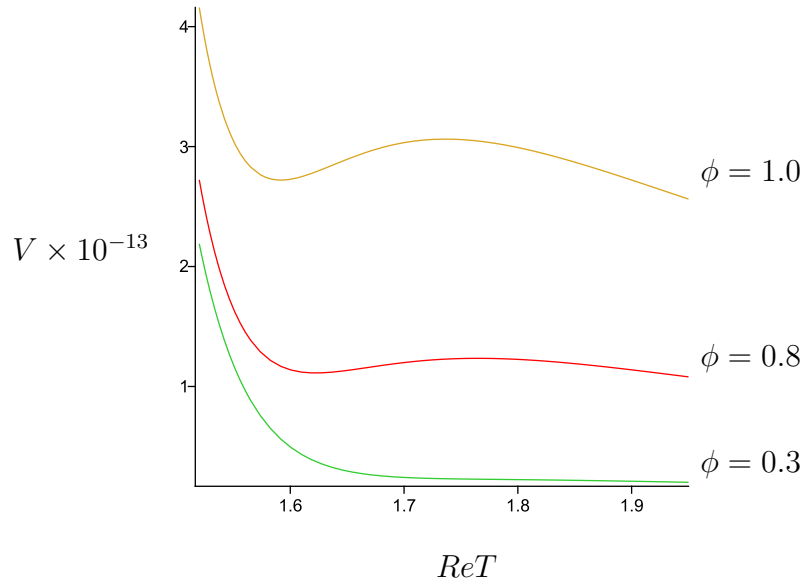


Figure 4: The same scalar potential as Figure 3. The top (yellow) curve corresponds to the slice of $\phi = 1.0$. Similarly, the second (red) curve and lowest (green) curve correspond to $\phi = 0.8$ and 0.3 , respectively.

Then, the values of superpotential and Hubble parameter vary like $W = \sqrt{N}m^2\phi(\alpha - 1)$ and $H = \sqrt{N}m^2(1 - \alpha)$. Thus, the condition that the moduli is stabilized during the waterfall is the same as Eq. (32). During the waterfall, however, ϕ rolls to the ISS point which is smaller than ϕ_* . Thus Eq. (32) would not be satisfied during the waterfall. The moduli would not be stabilized during the waterfall.

3.3 Waterfall by modulus

The inflation scenario in the previous section can be realized if the relation $\phi_* < \phi_c$ is satisfied. That requires that $N = \mathcal{O}(10^5)$. That might be too large to construct explicit (string) models. If the value of N is small like $N = \mathcal{O}(1 - 10)$ and the situation $\phi_* > \phi_c$ is realized, the modulus is destabilized before the waterfall by tachyonic masses of χ and $\bar{\chi}$. That corresponds to the lowest (green) curve in Figure 4. Then, the inflation ends. The inflation itself has no problem. That is, we can realize sufficiently large e-folding before the destabilization of the modulus T . However, once the modulus T is destabilized with the superpotential (24), the modulus T runs away to infinity.

We can avoid such a runaway behavior by adding the non-perturbative term Be^{bT} with the positive exponent b , i.e.,

$$W = W_{ISS} - Ae^{-aT} + Be^{bT}. \quad (37)$$

As said above, the previous scenario with $B = 0$ has no problem for the inflation to realize a sufficiently large value of e-folding. Hence, we take values of B and b such that they are small enough not to affect the inflation around T_{inf} , which denotes the value of T determined by $D_TW = 0$ with $B = 0$, i.e. Eq. (29).

In this case, the inflation terminates due to the instability caused by the moduli T , but not $\chi, \bar{\chi}$ fields. That is, the modulus becomes the waterfall field. That has new and interesting effects on several cosmological aspects. However, studies on such aspects are beyond the purpose of this paper. We would study them elsewhere. Figure 5 shows the scalar potential V of ReT and ϕ for the superpotential (37).

At any rate, the runaway of T to infinity is avoided by the term with the positive exponent, Be^{bT} . Then, the field ϕ rolls down around ϕ_c , where $\chi, \bar{\chi}$ fields become tachyonic. Then, the system approaches toward the ISS vacuum. At that point the dominant superpotential can be written by

$$W = w_0 - \mu^2\Phi_{aa} + Be^{bT} + \dots. \quad (38)$$

Similar to the discussion at the beginning of section 3.2, the modulus T is stabilized around $D_TW = 0$, where the modulus has the mass,

$$m_T \approx \frac{bw_0}{\sqrt{T + T^*}}. \quad (39)$$

The SUSY breaking is dominated by $F^\Phi = \mu^2$, while the F^T is suppressed as $F^T \simeq 3b^{-1}m_{3/2} \ll F^\Phi = \mu^2 \approx m_{3/2}$. Recall again that μ^2 and m^2 are independent each other. Thus, we would realize much smaller value of $m_{3/2}$ than H_{inf} .

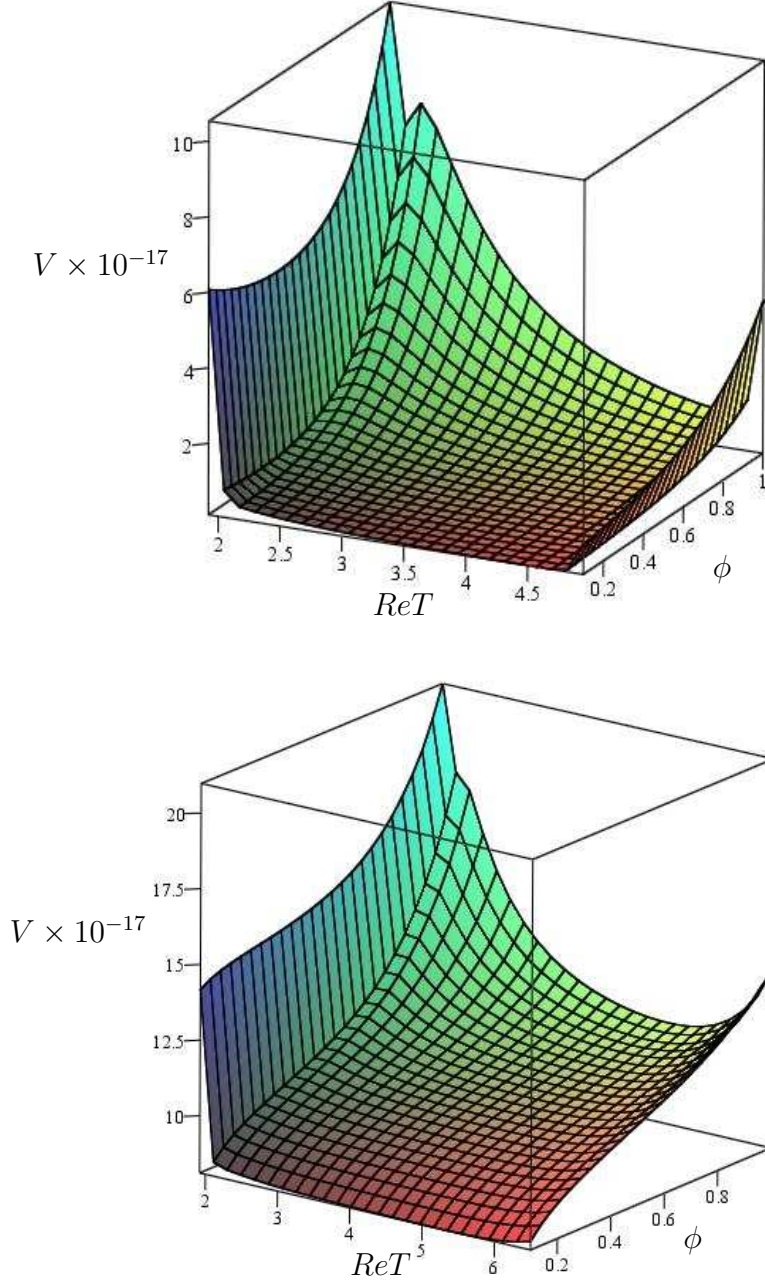


Figure 5: The potential including positive exponent term. We take the values of parameters as $b = 10, B = 10^{-30}$ in the upper figure and $b = 1, B = 10^{-10}$ in the lower figure. The other parameters are taken as the same values as those in Figures 3.

3.4 Positive exponent

It would be simple to consider the following superpotential only including W_{ISS} and a single modulus-dependent term with a positive exponent, i.e.,

$$W = W_{ISS} + Ae^{aT}, \quad (40)$$

where $A \ll 1$ and $a > 0$. During the inflation era, the dominant terms in the superpotential are written by

$$W = -m^2 TrY + Ae^{aT} + \dots. \quad (41)$$

We take the initial condition $\phi_e = 1$. Then, the inflation occurs. As far as ϕ is large enough, the modulus T is stabilized at $D_T W = 0$ for the above superpotential (41), i.e.

$$aT \sim -\log(m^2 TrY), \quad (42)$$

and the modulus T has the mass $m_T \simeq am^2 TrY / \sqrt{(T + \bar{T})}$. Thus, a sufficiently large value of e-folding can be realized, similar to that in the previous sections. As the field ϕ rolls down the scalar potential and its value becomes of $\mathcal{O}(0.1)$, the modulus mass becomes of $\mathcal{O}(H_{inf})$. The modulus shifts its value, but the modulus is not destabilized, because there are quite high potential barriers at both small and large values of ReT . Figure 6 shows the scalar potential of ReT at the slice of $\phi = 1.0, 0.7$ and 0.1 . Then, at $\phi = \phi_c$, the tachyonic direction appears along χ and $\bar{\chi}$, but not T . The system approaches toward the ISS vacuum, where the dominant terms in the superpotential can be written by

$$W = w_0 - \mu^2 \Phi_{aa} + Ae^{aT} + \dots. \quad (43)$$

At the ISS vacuum, the large modulus mass is realized and we can realize the gravitino mass $m_{3/2}$ smaller than H_{inf} .

4 Conclusion

We have studied the inflation scenario from the viewpoint of the moduli stabilization. We have proposed the scenario that the superpotential has a large value during the inflation era and it induces a large mass of the modulus field. Then, we could realize the inflationary expansion of the Universe with a sufficiently large value of e-folding N_e without destabilizing the modulus. We study our scenario by using a simple model, the deformed ISS inflation model. It is found that in that model a sufficiently large e-folding can be obtained during the inflation, but at the final stage the modulus may be destabilized. Then, the modulus would run away to infinity in the simple KKL'T superpotential. Thus, we would need another bump/barrier to avoid the runaway behavior of the modulus. Indeed, we have added the term with a positive exponent, although another type of potential bump may also be useful. As a result, this model shows a new aspect. The inflation is

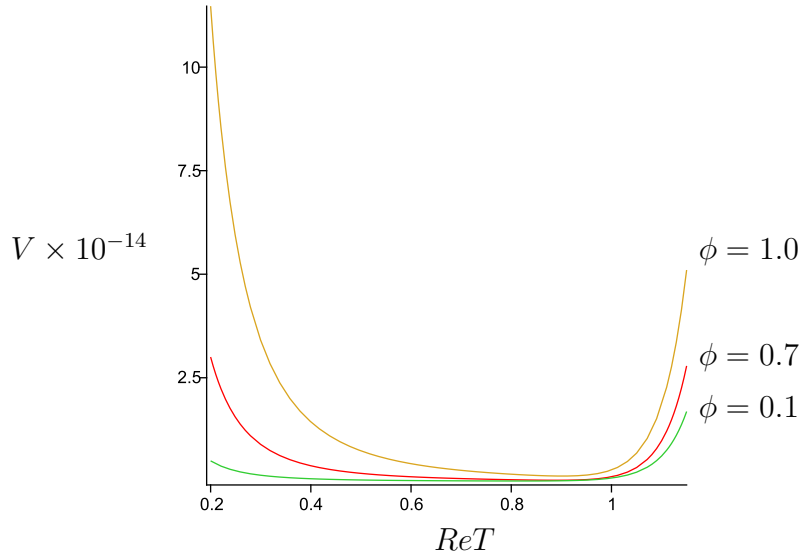


Figure 6: The potential of the positive exponent superpotential. We take the values of parameters as $w_0 = -10^{-14}$, $A = -10^{-12}$, $a = -10$, $m = 10^{-4}$, $N = 3$. The top (yellow), middle (red) and lowest (green) curves show that when $\phi = 1.0$, $\phi = 0.7$, $\phi = 0.1$.

terminated when the modulus mass becomes tachyonic. That is, the modulus becomes the waterfall field. That has new and interesting effects on several cosmological aspects. We would study them elsewhere. At any rate, the modulus is not destabilized by the term with a positive exponent. Then, the system approaches toward the ISS vacuum. At the true (ISS) vacuum with the modulus stabilized, we can obtain the gravitino mass $m_{3/2}$, which is much smaller than the Hubble parameter H_{inf} during the inflation.

On the other hand, other inflation models would lead to a different behavior. The modulus might be stabilized in the whole inflation process in another inflation model with a large value of superpotential. It would be interesting to apply our studies to several types of inflation models.

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